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ABOUT THE METHOD FOR SOLVING TWO- AND THREE-INDEX FUZZY TRANSPORTATION PROBLEMS

The purpose of the article is to develop the method for solving two- and three-index fuzzy transportation problems

The fuzzy models of transportation problems allow formalizing situation for the using of fuzzy resources which should take into account in case of the uncertainty in the determination of the volume of production and consumption. Additional information is introduced in these models about the possible values of the needs in the form of fuzzy sets. The corresponding membership functions can be viewed as a way to approximate an expert display of available non-formalized his ideas about the real value of the parameter on the basis of which the various possible values of each particular parameter values are assigned to the membership functions.

Research methods is a mathematical modeling based on transport problem, solved on a network, that consists of a finite number of nodes and arcs between them, is a linear programming problem (LPP), if the total cost of transport and restrictions on traffic volumes are defined by linear functions

Scientific novelty of the research is to solve transportation problems with intermediate points that reduce to solving two-index and three-index tasks are considered. The ways to find the optimal solution of fuzzy transportation problem, in which resources are given in the form of triangular fuzzy numbers.

Conclusions. Method of transformation in the system of constraints for the solving the crisp and fuzzy transportation problems with intermediate points has been proposed. The proposed method is illustrated by the example of real transportation problem.

Key words: transportation problem, fuzzy resources, membership function, triangular fuzzy numbers, optimization

Introduction. The main essence of the transportation problem (TP), which is one of the examples in mathematical programming problems, it is optimal distributing of the «producers» similar products among a group of «consumers» with the defined limits on the «supply» and «demand».

Description and study of transport-type tasks devoted a lot of scientific publications (Voloshyn and Mashchenko, 2010). This works contain informative formulations of different applied problems, which are reducing to transport problem, and construct their mathematical models.

Transport problem, solved on a network, that consists of a finite number of nodes and arcs between them, is a linear programming problem (LPP), if the total cost of transport and restrictions on traffic volumes are defined by linear functions. A typical problem is the transportation of products from *m* producers to *n* consumers.

Research results. The task is to determine the transport plan x_{ij}, $i=\overline{1,m}$, $j=\overline{1,n}$, in which demands of all consumers B_j, $j=\overline{1,n}$, will be fully satisfied, the entire volume production will be exported from the points of production A_i, $i=\overline{1,m}$, and total transportation costs are minimal (Yudin and Golshteyn, 1969). To do this, its need to determine a set of values $x_{ij} \geq 0$, $i=\overline{1,m}$, $j=\overline{1,n}$, that will satisfy such conditions

$$\sum_{j=1}^{n} x_{ij} = a_i \,,\; i=\overline{1,m}\,, \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \,,\; j=\overline{1,n}\,, \tag{2}$$

and they must make objective function

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \tag{3}$$

to reach a minimum value. Here a_i – the volume of production in station A_i, $i=\overline{1,m}$, b_j – consumption volume in station B_j, $j=\overline{1,n}$, c_{ij} – transport costs for delivery of production unit from station A_i to station B_j, $i=\overline{1,m}$, $j=\overline{1,n}$.

Efficient algorithms for solving the transportation problem (3), (1), (2) were created for problems, where the cost and consumption coefficients are known a priori. However, in practice, quite often are analyzed such examples in which these parameters may not be set accurately. For example, postage may be changed during transportation. Requests for the volume of consumption cannot be determined due to the nature of certain uncontrollable factors.

On the way of detalization and clarifying the parameters of the model (1)–(3) is used a description of the parameters with the help of fuzzy sets. Into the model is introduced additional information in the form of membership functions of this fuzzy sets. These functions can be viewed as a way to approximate an expert`s non-formalized notion about the real value of the parameter. The values of the membership functions are weight coefficients, which experts attribute to the different possible values of each particular parameter.

Definition 1. A fuzzy subset $\tilde{A}$ of the universal set X is the set of pairs $\tilde{A}=\{(\mu_{\tilde{A}}(x),x)\}$, where $\mu_{\tilde{A}}(x): X \to [0,1]$ is the mapping of the set X into the unit interval [0,1] which is called the membership function of fuzzy set (Bablu and Tapan, 2005).

The value of the membership function $\mu_{\tilde{A}}(x)$ for the item $x \in X$ is called the membership degree. Interpretation of the membership degree is a subjective measure of how the element $x \in X$ corresponds to the concept, the meaning of which is formalized by the fuzzy set $\tilde{A}$.

The usual sets $A_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}, \alpha \in [0,1]$ are called the sets of α – level (α – cuts) of a fuzzy set $\tilde{A}$.

Let choose as a universal set X a set of real numbers R^1, i.e. $X = R^1$.

Definition 2. Fuzzy trapezoidal number of $\tilde{A}$ is an ordered quadruple of numbers *(a, b, c, d)*, $a \leq b \leq c \leq d$, which determine membership function $\mu_{\tilde{A}}(x)$ of such views (Dubois, 1987):

1. $\mu_{\tilde{A}}(x)=\dfrac{x-a}{b-a}, x \in [a,b)$; 2. $\mu_{\tilde{A}}(x)=1, x \in [b,c]$; (T1)

3. $\mu_{\tilde{A}}(x)=\dfrac{d-x}{d-c}, x \in (c,d]$; 4. $\mu_{\tilde{A}}(x)=0, x \notin [a,d]$.

If we set $b=c$, we get a fuzzy number, which is called a triangular fuzzy number (a triplet).

Definition 3. Fuzzy triangular number of $\tilde{A}$ is an ordered triple of numbers *(a, b, c)*, $a \leq b \leq c$, which determine membership function $\mu_{\tilde{A}}(x)$ of such views (Dubois, 1987):

1. $\mu_{\tilde{A}}(x)=\dfrac{x-a}{b-a}, x \in [a,b]$; 2. $\mu_{\tilde{A}}(x)=\dfrac{c-x}{c-b}, x \in [b,c]$; 3. $\mu_{\tilde{A}}(x)=0, x \notin [a,c]$. (T2)

Corresponding to traditional TP fuzzy transportation problem (FTP) can be formally written as:

$$min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij} , \qquad (4)$$

with constraints

$$\sum_{j=1}^{n} x_{ij} = \tilde{a}_i, \quad i = \overline{1,m}, \tag{5}$$

$$\sum_{i=1}^{m} x_{ij} = \tilde{b}_j, \quad j = \overline{1,n}, \tag{6}$$

$$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j, \tag{7}$$

$$x_{ij} \geq 0, \quad i = \overline{1,m}, \; j = \overline{1,n}.$$

We will consider the transportation problem of fuzzy production and distribution of resources, the volumes of which are defined by triangular fuzzy numbers, $\tilde{a}_i = (a_i^l, a_i, a_i^r), i = \overline{1,m}$, $\tilde{b}_j = (b_j^l, b_j, b_j^r), j = \overline{1,n}$, where values of permissible variations $0 \leq a_i^l \leq a_i$, $a_i^r \geq 0$, $i = \overline{1,m}$, $0 \leq b_j^l \leq b_j$, $b_j^r \geq 0$, $j = \overline{1,n}$, determine the marginal changes of resources in model (5)–(7). Transportation costs of output delivery per unit c_{ij}, $i = \overline{1,m}$, $j = \overline{1,n}$, we will assume known in advance.

In this case, the transportation problem can be considered as a fuzzy linear programming problem with resource constraints in the form of triangular fuzzy numbers, the objective function of which has the form $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \rightarrow min$, and the limitations – of the form (5)–(7). The solution to this FTP can be found on the basis of the approach, proposed in (Zadeh, 1965).

Let Z_l и Z_u – the minimum and maximum value of the objective function Z at a given fuzzy set of resources (at $\lambda = 0$ and $\lambda = 1$, respectively), $L_1 = min(Z_l, Z_u)$, $U_1 = max(Z_l, Z_u)$. Taking into account these levels we can write the fuzzy problem of determining the variables $x_{ij} \geq 0$, $i = \overline{1,m}$, $j = \overline{1,n}$, which satisfying fuzzy constraints

$$Z \leq \tilde{s}, \; \tilde{s} = (L_1, L_1, U_1), \tag{8}$$

$$\sum_{j=1}^{n} x_{ij} = \tilde{a}_i, \quad i = \overline{1,m}, \; \sum_{i=1}^{m} x_{ij} = \tilde{b}_j, \quad j = \overline{1,n}, \; \sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j. \tag{9}$$

Membership functions of fuzzy sets of constraints (8), (9) can be written as:
for a first constraint (8)

$$\mu^1\left(\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}\right)=\begin{cases}1, \quad npu \sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}<L_1,\\ \left(U_1-\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}\right)\Big/(U_1-L_1), \quad npu\ L_1\le\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}<U_1,\\ 0, \quad npu \sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}\ge U_1,\end{cases}$$

for the *i* constraint (9), $i=\overline{1,m}$,

$$\mu_i^2\left(\sum_{j=1}^{n}x_{ij}\right)=\begin{cases}0, \quad npu \sum_{j=1}^{n}x_{ij}<a_i-a_i^l,\\ \left(\sum_{j=1}^{n}x_{ij}-a_i+a_i^l\right)\Big/a_i^l, \quad npu\ a_i-a_i^l\le\sum_{j=1}^{n}x_{ij}<a_i,\\ \left(a_i+a_i^l-\sum_{j=1}^{n}x_{ij}\right)\Big/a_i^r, \quad npu\ a_i\le\sum_{j=1}^{n}x_{ij}<a_i+a_i^r,\\ 1, \quad npu \sum_{j=1}^{n}x_{ij}\ge a_i+a_i^r,\end{cases}$$

for the *j* constraint (9), $j=\overline{1,n}$,

$$\mu_j^3\left(\sum_{i=1}^{m}x_{ij}\right)=\begin{cases}0, \quad npu \sum_{i=1}^{m}x_{ij}<b_j-b_j^l,\\ \left(\sum_{i=1}^{m}x_{ij}-b_j+b_j^l\right)\Big/b_j^l, \quad npu\ b_j-b_j^l\le\sum_{i=1}^{m}x_{ij}<b_i,\\ \left(b_j+b_j^r-\sum_{i=1}^{m}x_{ij}\right)\Big/b_j^r, \quad npu\ b_i\le\sum_{i=1}^{m}x_{ij}<b_j+b_j^r,\\ 1, \quad npu \sum_{i=1}^{m}x_{ij}\ge b_j+b_j^r.\end{cases}$$

Fuzzy values of resource in constraints (9) can be written as $\tilde{a}_i=(a_i-\lambda a_i^l,a_i,a_i+\lambda a_i^r),i=\overline{1,m},\ \ \tilde{b}_j=(b_j-\lambda b_j^l,b_j,b_j+\lambda b_j^r),\ j=\overline{1,n}$. Then, based on Bellman-Zadeh`s definition of a fuzzy solution problem (8), (9) can be rewritten in the form

$$\max\ \lambda \tag{10}$$

with constraints

$$\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}-\lambda(U_1-L_1)\ge L_1,$$

$$\sum_{j=1}^{n} x_{ij} - \lambda a_i^l \ge a_i - a_i^l, \quad \sum_{j=1}^{n} x_{ij} + \lambda a_i^r \le a_i + a_i^r, \tag{11}$$

$$\sum_{i=1}^{m} x_{ij} - \lambda b_j^l \ge b_j - b_j^l, \; \sum_{i=1}^{m} x_{ij} + \lambda b_j^r \le b_j + b_j^r,$$

$$\sum_{i=1}^{m}(a_i - a_i^l + \lambda a_i^l) = \sum_{j=1}^{n}(b_j - b_j^l + \lambda b_j^l),$$
$$\sum_{i=1}^{m}(a_i + a_i^r - \lambda a_i^r) = \sum_{j=1}^{n}(b_j + b_j^r - \lambda b_j^r), \tag{12}$$

$$0 \le \lambda \le 1, \; x_{ij} \ge 0, \; i = \overline{1,m}, \;\; j = \overline{1,n}.$$

It is obvious that this problem is a classic LP problems, for finding solutions of which you can apply any variant of the simplex method.

To suppose, that there are points of production, intermediate points and points of uniform product consumption. There are specified the maximum possible volumes of production of the product in every point of production, the minimum acceptable levels of consumption by each point of consumption, restrictions on the volume of transportation of the product from each production point to each intermediate point, restrictions on the volume of transportation of the product from each intermediate point to each point of consumption. We need to find transportation plan, which will ensure the effective functioning of the system and satisfy points restrictions on possible produced, consumed and transmitted volumes of uniform product.

Let set *I* as the set of production points, *J* as the set of intermediate points, K as the set of consumption points, A_i, $i = \overline{1,I}$, as the maximum volume of product manufacturing by i point of production; B_k, $k = \overline{1,K}$, as minimum acceptable volume of the product, which must be delivered to the point of consumption *k*; C_{ij}, $i = \overline{1,I}$, $j = \overline{1,J}$, as maximum volume of product, that can be delivered from the point of production i to intermediate point *j*; D_{jk}, $j = \overline{1,J}$, $k = \overline{1,K}$, as maximum volume of product, that can be delivered from an intermediate point j to consumption point *k*; x_{ijk}, $i = \overline{1,I}$, $j = \overline{1,J}$, $k = \overline{1,K}$, as volume of product, that will be transported from the production point i through the intermediate point j to consumption point *k*.

General mathematical model of transportation problem of uniform product is a system of following restrictions

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} \le A_i, \; i = \overline{1,I}, \tag{13}$$

(volume of product manufacturing by production point must not exceed the maximum possible volume);

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk} \geq B_k , k = \overline{1,K} , \qquad (14)$$

(point of consumption must get the volume of product, which is not be below the minimum amount);

$$\sum_{k=1}^{K} x_{ijk} \leq C_{ij} , \ i = \overline{1,I} , \ j = \overline{1,J} , \qquad (15)$$

(transporting volume of product from the point of production to an intermediate point must not exceed the maximum allowable volume);

$$\sum_{i=1}^{I} x_{ijk} \leq D_{jk} , j = \overline{1,J} , \ k = \overline{1,K} , \qquad (16)$$

(transporting volume of product from the intermediate point to the point of consumption must not exceed the maximum allowable volume);

$$x_{ijk} \geq 0 , \ i = \overline{1,I} , \ j = \overline{1,J} , \ k = \overline{1,K} , \qquad (17)$$

(natural restrictions on variables).

As the optimality criteria for determining the efficiency of functioning the system, which may depend on various parameters of the sought-for traffic plan, can be considered the cost parameters of the plan. Let set a_i , $i = \overline{1,I}$, as cost price of manufacturing of unit of product by production point *i*; b_k , $k = \overline{1,K}$, as selling price of a unit of product to consumption point *k*; c_{ij} , $i = \overline{1,I}$, $j = \overline{1,J}$, as cost of delivery of unit of product from production point i to intermediate point *j*; d_{jk} , $j = \overline{1,J}$, $k = \overline{1,K}$, as cost of delivery of unit of product from the intermediate point *j* to consumption point *k*.

The problem of cost minimizing is a problem of determining of the plan of transportation x_{ijk} , $i = \overline{1,I}$, $j = \overline{1,J}$, $k = \overline{1,K}$, which satisfies constraints (13)–(17) , and make criteria

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(a_i + c_{ij} + d_{jk})x_{ijk} \to \min . \qquad (18)$$

to receive it`s optimum value.

The problem of income maximization is a problem of determining of the plan of transportation x_{ijk} , $i = \overline{1,I}$, $j = \overline{1,J}$, $k = \overline{1,K}$, which satisfies constraints (13)–(17), and make criteria

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} b_k x_{ijk} \to \max . \qquad (19)$$

to receive it`s optimum value.

The problem of profit maximization is a problem of determining of the plan of transportation x_{ijk}, $i=\overline{1,I}$, $j=\overline{1,J}$, $k=\overline{1,K}$, which satisfies constraints (13)–(17), and make criteria

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} (b_k - a_i - c_{ij} - d_{jk}) x_{ijk} \to \max . \qquad (20)$$

to receive it`s optimum value.

As plan indexes we can consider indexes, which are based on preferences, which are defined by controllable elements in system. Let points of consumption be the controllable elements of system. With each of the points of consumption $k=\overline{1,K}$ we will connect preference function $\phi_k : R \to R,\ k=\overline{1,K}$, which determines the preference of the consumer on the volume of consumed product. Note, that volume of product, consumed by an element $k = \overline{1,K}$, defined as $\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk}$, $k=\overline{1,K}$. In practice, consumers find it difficult to determine their preferences with respect to each of possible volume of the product consumed by them. One way to specify such preferences is a method of ranking intervals of corresponding volumes of product. The formalization of such preferences is linked to the definition of the final inserted sequence of intervals of corresponding volumes of product. Let each of consumers sets *p*+1 inserted intervals $[B_{kl}^{-}, B_{kl}^{+}], l=\overline{0,p}$, $k=\overline{1,K}$. Inserting of intervals means that the following conditions $[B_{kl}^{-}, B_{kl}^{+}] \subseteq [B_{kl+1}^{-}, B_{kl+1}^{+}], l=\overline{0,p-1},\ k=\overline{1,K}$ are true. So interval $[B_{k0}^{-}, B_{k0}^{+}]$ is the most preferred to consumer k, $k=\overline{1,K}$, and interval $[B_{k1}^{-}, B_{k1}^{+}]$ is less preferred, etc. Then preferences functions are defined as piecewise constant functions of the following form:

$$\phi_k(u) = \begin{cases} 0, & if\ u \in [B_{k0}^{-}, B_{k0}^{+}], \\ l, & if\ \exists\, l \in \{1,...,p\} : u \in [B_{kl}^{-}, B_{kl}^{+}], u \notin [B_{kl-1}^{-}, B_{kl-1}^{+}], \\ p+1, & if\ u \notin [B_{kp}^{-}, B_{kp}^{+}], \end{cases} \qquad (21)$$

where $u \in R,\ k=\overline{1,K}$. Then the problem of choosing the most preferred transportation plan is set as the following multi-criteria optimization problem: it is necessary to determine the transportation plan x_{ijk}, $i=\overline{1,I}$, $j=\overline{1,J}$, $k=\overline{1,K}$, which satisfies constraints (13) – (17) and make criteria, which define consumer points preferences

$$\phi_k(\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk}) \to \min,\ k=\overline{1,K}\cdot \quad (22)$$

to receive it`s optimum values.

If the set of consumers is ordered in terms of their priority in determining the efficiency of system as a whole, in solving multi-criteria optimization problem (13)–(17), (22) may be used lexicographical compromise scheme (Zimmermann, 1992). In the case of equivalence of consumers as a compromise scheme for solving the problem of multi-criteria optimization (13)–(17), (22) may be used maximin convolution (Zimmermann, 1992).

In situations, where consumers can't formalize a system of inserted intervals of values of volumes of necessary product, we will use a fuzzy approach for specifying the quantities of resources required.

Let sets $[b_k, c_k]$, $k=\overline{1,K}$, as the most preferred ranges of product volumes for consumers k, $k=\overline{1,K}$. Permissible variation in required by consumers volumes of product will be set as values $a_k \le b_k$ and $c_k \le d_k$, $k=\overline{1,K}$, and quantities a_k, d_k, $k=\overline{1,K}$, are, respectively, the smallest and largest values of necessary volumes of resource. This allows to formalize values of requires in a form of trapezoidal fuzzy numbers $\tilde{v}_k = (a_k, b_k, c_k, d_k)$, $k=\overline{1,K}$. Assuming linear character of membership functions $\mu_{\tilde{v}_k}(x)$, $k=\overline{1,K}$, with form (T2), we obtain a fuzzy transportation problem with intermediate points and constraints on volumes of resource consumption in the form of inequalities with fuzzy right side

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk} \ge \tilde{v}_k^-,\ \sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk} \le \tilde{v}_k^+,\ k=\overline{1,K}, \quad (23)$$

where $\tilde{v}_k^- = (a_k, b_k, b_k)$, $\tilde{v}_k^+ = (c_k, c_k, d_k)$, $k=\overline{1,K}$, – fuzzy triangular numbers, that determine the minimum and maximum requirements of product.

The solution of obtained optimization problem of resource allocation, taking into account any of criterion functions (18)–(20) and system of constraints (13), (23), (15)–(17) can be obtained based of general approach, set out above.

The proposed above models of transport problems allow solving the problem of distributing the limited capacities of data transmission channels between various nodes of the Internet service provider's network. Suppose that there is a local computer network of the enterprise (higher education institution) that provides access to the Internet network for users. Access of users to the global network and obtaining the necessary information is made by means of several communication servers located on the territory of the information and computing center of the enterprise and connected by high-speed external communication channels with Internet providers. Server bandwidth levels lie within the bandwidth of the local network (for example, 1GB per second). Let's assume that the needs of network subscribers are known in increasing the speed of obtaining a certain amount of information. The wishes (preferences) of subscribers regarding possible volumes of increase in capacity for transferring information from the provider to the user node are specified. To implement the wishes, it is necessary to update the capacity of the switching servers of the network by deploying new, more powerful computers or by increasing the number of existing servers. In this case, the value of the total server capacity, both in case of increasing the capacity of the existing computer fleet, and in the case of increasing the number of servers is assumed to be the same.

When solving this transportation problem for different number of servers and 17 user's connections the following results were obtained: when using 2 identical communication servers with a total capacity of 3 Gb/s, the capacity of local connections is 259, 159, 149, 166, 273, 115, 163, 274, 152, 148, 125, 144, 90, 365, 180, 89, 149 Mb / s, which coincided with the previous decisions. With the use of 3 communicators with a total capacity of 3 Gb/s, the capacity of local connections is 260, 148, 146, 190, 258, 114, 175, 266, 146, 195, 124, 143, 90, 335, 180, 89, 141 Mb/s (also coincided with previous decisions). With the increase in the maximum values of the throughput of local connections to 280, 180, 170, 200, 290, 125, 190, 290, 170, 210, 135, 160, 100, 390, 195, 95, 165 Mb / s in case of using two communication. Optimum speeds of local connections equal to 128, 161, 160, 124, 284, 124, 152, 287, 165, 208, 134, 158, 100, 378, 194, 94, 149 Mb/s were obtained with the total throughput of 3 Gb/s, respectively, and in the case of using 3 communicators, the values of local connections 271, 146, 153, 65, 273, 123, 123, 284, 162, 206, 131, 158, 97, 378, 192, 92, 146 Mb/s, respectively.

Conclusions. A wide class of applied problems of resource allocation belongs to the class of multi-index problems. In this article is offered the method of solving of two-index and three-index traditional and fuzzy transportation problems. This method is used for case of fuzzy constraints of resources.

The fuzzy models of transportation problems allow to formalize situation for the using of fuzzy resources which should take into account in case of the uncertainty in the determination of the volume of production and consumption. In these models is introduced additional information about the possible values of the needs in the form of fuzzy sets. The corresponding membership functions can be viewed as a way to approximate an expert display of available non-formalized his ideas about the real value of the parameter on the basis of which the various possible values of each particular parameter values are assigned to the membership functions. There are used the triangular and trapezoidal fuzzy numbers.

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ПРО МЕТОД РОЗВ'ЯЗКУ ДВО- ТА ТРЬОХФАЗНОЇ ТРАНСПОРТНОЇ ЗАДАЧІ

Мета дослідження – розробка методу розв'язання задач нечіткого транспорту з двома та три індексами

Нечіткі моделі транспортних задач дозволяють формалізувати ситуацію для використання нечітких ресурсів, які слід враховувати в разі невизначеності у визначенні обсягу виробництва та споживання. У цих моделях вводиться додаткова інформація про можливі значення потреб у вигляді нечітких множин. Відповідні функції приналежності можна розглядати як спосіб наближення експертного відображення доступних неформалізованих його уявлень про реальне значення параметра, на основі якого для різних функцій приналежності призначаються різні можливі значення кожного конкретного значення параметра.

Методи дослідження – це математичне моделювання, засноване на транспортній задачі, вирішеній у мережі, яка складається з кінцевого числа вузлів та дуг між ними, це проблема лінійного програмування (LPP), якщо загальна вартість транспорту та обмеження на обсяги трафіку визначається лінійними функціями

Наукова новизна дослідження – це вирішення проблем транспорту з проміжними точками, що зводяться до вирішення двох-індексних та три-індексних завдань. Наведено шляхи пошуку оптимального рішення задачі нечіткої транспорту, в якій даються ресурси у вигляді трикутних нечітких чисел.

Висновки. Запропоновано метод перетворення системи обмежень для вирішення проблем з чітким та нечітким транспортом з проміжними точками. Пропонований спосіб ілюструється на прикладі проблеми реального транспортування.

Ключові слова: проблема транспортування, нечіткі ресурси, функція приналежності, трифазны нечіткі числа, оптимізація.

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О МЕТОДЕ РЕШЕНИЯ ДВУХ- И ТРЕХФАЗНОЙ ТРАНСПОРТНОЙ ЗАДАЧИ

Целью исследования является разработка метода решения двух- и трехиндексных задач нечеткого переноса

Нечеткие модели транспортных проблем позволяют формализовать ситуацию для использования нечетких ресурсов, которые должны учитываться в случае неопределенности в определении объема производства и потребления. В этих моделях вводится дополнительная информация о возможных значениях потребностей в виде нечетких множеств. Соответствующие функции принадлежности можно рассматривать как способ аппроксимировать экспертное отображение доступных неформализованных его представлений о реальном значении параметра, на основе которого различные функциональные значения каждого значения параметра присваиваются функциям членства.

Методы исследования – это математическое моделирование, основанное на транспортной проблеме, решаемой в сети, состоящей из конечного числа узлов и дуг между ними, является задачей линейного программирования (LPP), если общая стоимость транспорта и ограничения на объемы трафика определенных линейными функциями

Научная новизна исследования – решение транспортных проблем с промежуточными точками, которые сводятся к решению двухиндексных и трехиндексных задач. Приводятся способы нахождения оптимального решения задачи нечеткого переноса, в котором представлены ресурсы в виде треугольных нечетких чисел.

Выводы. Предложен метод преобразования системы ограничений для решения четких и нечетких задач переноса с промежуточными точками. Предлагаемый метод иллюстрируется примером реальной транспортной проблемы.

Ключевые слова: проблема транспортировки, нечеткие ресурсы, функция принадлежности, трехфазные нечеткие числа, оптимизация.